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Nucleon–Nucleon Interaction in the Three-Nucleon System

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The three-nucleon system is reconsidered. The Faddeev equations are given leading to a set of integral equations. Solving these integral equations, suitable forms are considered for the nucleon–nucleon interaction. In the bound state of three-nucleon system, the form of the nuclear forces from the nucleon–nucleon interaction is important. In the present calculations, we consider the nuclear forces resulting from the nucleon–nucleon interaction by the exchange of a scalar meson, a pseudoscalar meson, and a massless vector meson. With this different meson exchange nucleon–nucleon interaction, the binding energy of the three-nucleon system is calculated by solving the Faddeev integral equations giving a value of 8.452 MeV.

I. INTRODUCTION

The three-nucleon system has been found as one of the most interesting systems in studying the static properties of nuclei. One of these properties is the nuclear forces, and the nucleon–nucleon interaction. The three-body problem has been solved successfully by Faddeev (1960, 1961, 1962). The Faddeev technique is more exact than the variational treatment of the problem, but it appears as the truncation of the two-body reaction matrix by one (or more finite number) pole term. This approximation is discussed by Lovelace (1964) on the basis of theoretical considerations. In this solution, Faddeev (1960, 1961, 1963, 1965) has shown that a well-behaved set of three-body equations involve the two-body T matrix rather than the potential. Consequently, the T matrix plays a central role in this approach.

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This means that in the three-body Faddeev equation, the two-body T matrix plays the part of a potential in the two-body Lippmann–Schwinger equation. Many authors[2] have shown that the Faddeev equations are reduced to a set of coupled, one-dimensional integral equations by using the separable (Harrington, 1966; Fulco and Wong, 1968; Osman, 1971, 1978a, b) two-body potentials. While for the case of local (Kharchenko et al., 1968; Phillips, 1968; Kok et al., 1968; Levinger, 1969; Osman, 1979a) potentials, the problem becomes difficult since the Faddeev equations are reduced to a set of coupled integral equations in two continuous variables. For all these potentials, the Faddeev equations remain a well-defined system whatever the potential form is.

The binding energy of the three-nucleon system has been widely considered. In calculating the binding energy of ^3H and ^3He, nonlocal separable potentials have been widely used (Lovelace, 1964; Mitra et al., 1976; Zingl et al., 1978; Harrington, 1966; Fulco and Wong, 1968; Osman, 1970, 1978, 1979a, b, 1980). These separable potentials are taken as a central potential containing both attraction and repulsion parts (Harrington, 1966; Fulco and Wong, 1968; Osman, 1970) as well as containing tensor forces (Osman, 1978c, 1979a, b, 1980). Local potentials have been used (Kharchenko et al., 1968; Phillips, 1968; Kok et al., 1968; Levinger, 1969; Osman, 1979a) also in calculating the three-nucleon binding energy and it is taken to consist of attraction and repulsion parts.

In the present work, we calculate the binding energy of the three-nucleon system. We follow the Faddeev formalism. Faddeev equations are given as a well-defined set of coupled integral equations. The nucleon–nucleon interaction which we use in the present work is taken as the exchange of a scalar meson, a pseudoscalar meson and a massless vector meson. The equations developed for the interactions of these two spin-half nucleons are applied through the exchange of massive scalar and pseudoscalar mesons. For the low-energy nucleon–nucleon interaction, the difficulties from the intrinsic nature of the strong interactions are little in comparison with the unfortunate fact that the pion is a pseudoscalar particle. This difficulty is overcome by considering two-pion exchange which is equivalent to the exchange of a scalar meson of distributed mass.

In Section 2, integral equations are introduced for the nucleon–nucleon interaction with the exchange of a scalar meson, a pseudoscalar meson, and a massless vector meson. Calculations and results are given in Section 3. Section 4 is left for discussion and conclusions.

[2]For a review see Mitra et al. (1976); for another recent review see Zingl et al. (1978).

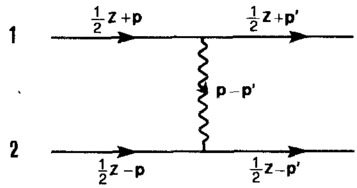

Fig. 1. Feynman diagrams in second order. The solid lines are the nucleons. The wavy lines are the mesons.

2. NUCLEON–NUCLEON INTERACTION WITH MESON EXCHANGE

In this section, we consider the nucleon–nucleon interaction with meson exchange. The nucleons are considered as particles 1 and 2 each with mass M. While the meson exchanged between the nucleons in successive processes is taken with a mass m, where $m \ll M$. The Feynman representation for the perturbation series of the nucleon–nucleon interaction by the successive exchange of the mesons are represented diagrammatically in Figure 1. The explicit considerations are given in Figure 2 for the fourth

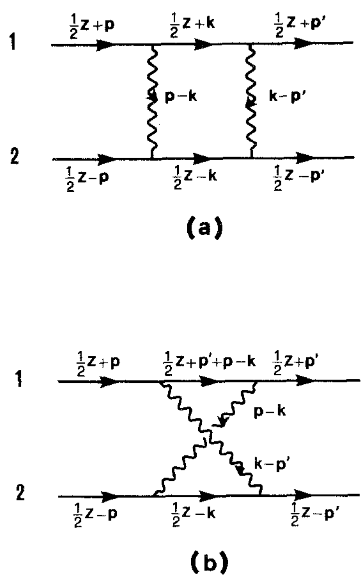

Fig. 2. Feynman diagrams in fourth order. The solid lines are the nucleons. The wavy lines are the mesons.

order, and in Figure 3 for the sixth order. The solid lines are the nucleons and the wavy lines are the mesons. The scattering amplitudes of these diagrams are given in integral equations. As an example, the scattering amplitude in second order as shown in Figure 1 is given by

$$T^{(2)} = -g^2 mM/\left[m^2 - (p-p')^2\right] \tag{1}$$

where g is the dimensionless coupling constant. This expression given by equation (1) is the usual nonrelativistic first Born approximation. The second Born approximation is given by the fourth-order diagrams shown in Figure 2.

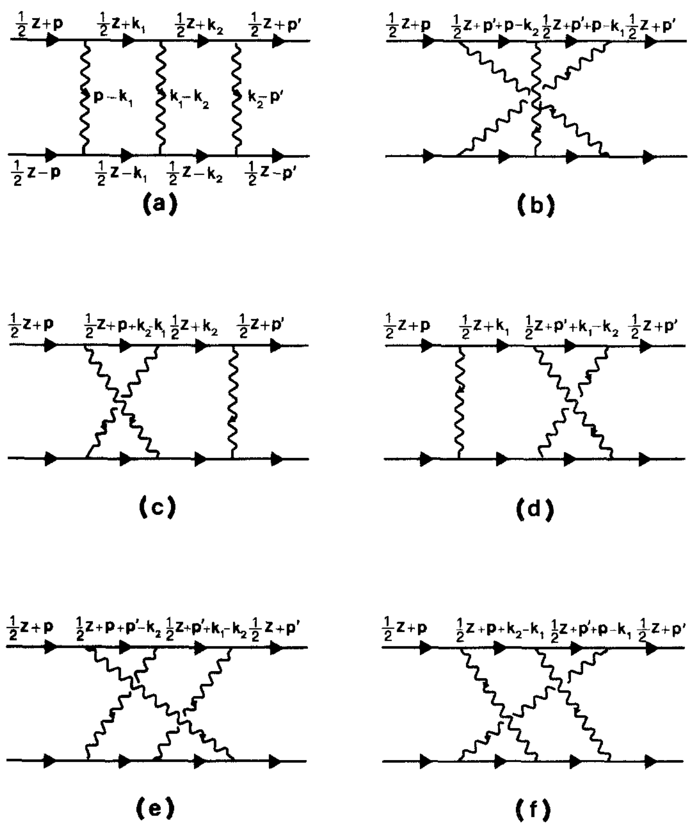

Fig. 3. Feynman diagrams in sixth order. The solid lines are the nucleons. The wavy lines are the mesons.

In the present work, we are interested in nucleon–nucleon interaction with meson exchange. We study here the exchange of a scalar meson, a pseudoscalar meson, and a massless vector meson.

(i) Scalar–Meson Exchange. If we restrict ourselves to the cases that the external particles are on the mass shell, then the scattering amplitude for the second-order $T^{(2)}_{\mathrm{sc-m}}$ which is the first Born approximation is given by the same expression given by equation (1). For the fourth-order diagrams shown in Figure 2 which include the second Born approximation, the scattering amplitude $T^{(4a)}_{\mathrm{sc-m}}$ representing Figure 2a is given by

$$T^{(4a)}_{\mathrm{sc-m}}=\frac{4ig^4M^4}{(2\pi)^4}\int\frac{d^4k}{\left[(M^2+\mathbf{k}^2)^{1/2}-(M^2-\mathbf{k}^2)^{1/2}\right]^2-k_0^2-i\epsilon}$$

$$\times\frac{1}{\left[(M^2+\mathbf{k}^2)^{1/2}+(M^2-\mathbf{k}^2)^{1/2}\right]^2-k_0^2-i\epsilon}$$

$$\times\frac{1}{\left[m^2+(\mathbf{p}-\mathbf{k})^2-k_0^2-i\epsilon\right]^2}$$

$$+\frac{g^4}{4(2\pi)^3}\int\frac{d^3k}{\left\{m^2+(\mathbf{p}-\mathbf{k})^2-\left[(M^2+\mathbf{k}^2)^{1/2}-(M^2-\mathbf{k}^2)^{1/2}\right]^2\right\}^2}$$

$$\times\frac{1}{\left\{m^2+(\mathbf{p}-\mathbf{k})^2-\left[(M^2+k^2)^{1/2}+(M^2-k^2)^{1/2}\right]^2\right\}^2}$$

$$\times\left\{\frac{1}{\left[m^2+(\mathbf{p}-\mathbf{k})^2\right]^{1/2}}\left[-8(\mathbf{p}\cdot\mathbf{k})\mathbf{k}^2+4\mathbf{k}^2(m^2-\mathbf{p}^2)\right.\right.$$

$$\left.-4M^2(\mathbf{p}-\mathbf{k})^2+12(\mathbf{p}\cdot\mathbf{k})^2-4m^2M^2\right]$$

$$-\frac{1}{(M^2+\mathbf{k}^2)^{1/2}}\left[-8(\mathbf{p}\cdot\mathbf{k})\mathbf{k}^2+4k^2(m^2-\mathbf{p}^2-M^2)\right.$$

$$\left.\left.+4(\mathbf{p}\cdot\mathbf{k})^2+4m^2M^2\right]\right\}. \tag{2}$$

For the second part of the fourth order given by the crossed-box diagram represented by Figure 2b, it gives a contribution which is smaller than that given by the uncrossed-box diagram. This is because in the case described by Figure 2b, it requires the existence of two mesons at the same time, which means that the nucleons must be farther off their mass shell. Neglecting retardation, for the case when the interaction is instantaneous, the contribution from the crossed-box is very small and it may be suppressed. The diagram in Figure 2b gives

$$T^{(4b)} = \frac{ig^4 m^2 M^2}{(2\pi)^4} \int \frac{d^4k}{M^2 + (2\mathbf{p}-\mathbf{k})^2 - \left[(M^2-\mathbf{k}^2)^{1/2} - k_0\right]^2 - i\epsilon}$$
$$\times \frac{1}{M^2 + \mathbf{k}^2 - \left[(M^2-\mathbf{k}^2)^{1/2} - k_0\right]^2 - i\epsilon}$$
$$\times \frac{1}{\left[m^2 + (\mathbf{p}-\mathbf{k})^2 - k_0^2 - i\epsilon\right]^2} \tag{3}$$

(ii) Pseudoscalar–Meson Exchange. In this case for nucleon–nucleon interaction with pseudoscalar–meson exchange, if we restrict ourselves to the case of scattering in the forward direction, then the second-order scattering amplitude $T^{(2)}_{\mathrm{ps-m}}$ vanishes because of some off-diagonal nature present in the matrix. Thus,

$$T^{(2)}_{\mathrm{ps-m}} = 0 \tag{4}$$

For the fourth-order terms, we have for the scattering amplitude arising from the uncrossed-box shown by Figure 2a, the expression

$$T^{(4a)}_{\mathrm{ps-m}} = \frac{ig^4}{(2\pi)^4} \int \frac{d^4k\left[M - \sigma_1 \cdot \left(\tfrac{1}{2}Z + k\right)\right]}{\left[(M^2+\mathbf{k}^2)^{1/2} - (M^2-\mathbf{k}^2)^{1/2}\right]^2 - k_0^2 - i\epsilon}$$
$$\times \frac{\left[M - \sigma_2 \cdot \left(\tfrac{1}{2}Z - k\right)\right]}{\left[(M^2+\mathbf{k}^2)^{1/2} + (M^2-\mathbf{k}^2)^{1/2}\right]^2 - k_0^2 - i\epsilon}$$
$$\times \frac{1}{\left[m^2 + (\mathbf{p}-\mathbf{k})^2 - k_0^2 - i\epsilon\right]^2} \tag{5}$$

This expression is dealt with in detail and by neglecting terms of order p/M, we get

$$T^{(4a)}_{\mathrm{ps-m}} = -\frac{g^4}{32\pi^2 M^2}\cdot\left[\ln\left(\frac{M}{m}\right)-1\right] \tag{6}$$

which has a large contribution, and so the full interaction must be treated.

(iii) Massless Vector–Meson Exchange. For the case of nucleon–nucleon interaction with a massless vector–meson exchange, the scattering amplitude for the fourth-order diagrams of the uncrossed-box represented by Figure 2a is given by

$$T^{(4a)}_{\mathrm{mv-m}} = \frac{ig^4}{(2\pi)^4}\int \frac{d^4k}{\left[(\mathbf{k}-\mathbf{p})^2 - i\epsilon\right]\left[(\mathbf{k}-\mathbf{p}')^2 - i\epsilon\right]}$$
$$\times \frac{1}{\left[M^2+\mathbf{k}^2-\left(\frac{1}{2}Z+k_0\right)^2-i\epsilon\right]\left[M^2+\mathbf{k}^2-\left(\frac{1}{2}Z-k_0\right)^2-i\epsilon\right]} \tag{7}$$

The whole contribution of this type of interaction will be clear, if we add the contributions coming from the fourth-order diagrams of the crossed-box and shown in Figure 2b. By adding the contributions from Figure 2b to that given by equation (7), we get

$$T^{(4)}_{\mathrm{mv-m}} = \frac{2ig^4}{(2\pi)^4}\int \frac{d^4k}{\left[(\mathbf{k}-\mathbf{p})^2 - i\epsilon\right]\left\{M^2+\mathbf{k}^2-\left[(M^2-\mathbf{k}^2)^{1/2}-p_0-k_0\right]^2-i\epsilon\right\}}$$
$$\times \frac{1}{M^2-(2\mathbf{p}-\mathbf{k})^2-\left[(M^2-\mathbf{k}^2)^{1/2}+p_0-k_0\right]^2-i\epsilon}$$
$$\times \frac{1}{M^2+\mathbf{k}^2-\left[(M^2-\mathbf{k}^2)^{1/2}+p_0-k_0\right]^2-i\epsilon} \tag{8}$$

3. CALCULATIONS AND RESULTS

From these scattering amplitudes introduced in Section 2, we can get the corresponding bound-state equations. Since all these equations can be written in a form as

$$T(p,p',Z)=B(p,p',Z)+\frac{i}{(2\pi)^4}\int d^4kB(p,k,Z)$$
$$\times K(Z,k)T(k,p',Z) \tag{9}$$

$B(p, p', Z)$ is the interaction kernel, while $K(Z, k)$ is the corresponding free two-body propagator. Defining the two-body bound-state amplitudes, then, we could proceed to the three-body problem.

We are interested in the present work with the three-nucleon bound-state. For the present case of the three-nucleon system, we follow the Faddeev formalism. The Faddeev equations for this three-nucleon bound-state have been widely considered. The resulting three-body integral equations are given explicitly (Harrington, 1966; Fulco and Wong, 1968; Osman 1970, 1971, 1978a–c; 1979a,b; 1980; Kharchenko et al., 1968; Phillips, 1968; Kok et al., 1968; Levinger, 1969). These equations are applied (Osman, 1970, 1978c, 1979a, b, 1980) for different potentials.

In the present work, we use the Faddeev equations which are given explicitly (Osman, 1971, 1978a,b) together with the two-body interactions considered in the present work.

Performing huge numerical calculations, we calculate the three-nucleon binding energy. We obtained for the bound-state three nucleons, a numerical value of 8.452 MeV. This value is very close to the experimental value of 8.48 MeV. Thus the present theoretically calculated value for the three-nucleon binding energy differs from the experimental value with a percentage of 0.9213%.

4. DISCUSSION AND CONCLUSIONS

It is very interesting to calculate the three-nucleon binding energy using a realistic potential. The nucleon–nucleon interaction considered in the present work is the meson exchange potential. Different meson exchange processes are considered. Calculations are performed for the nucleon–nucleon interaction with the exchange of a scalar meson, a pseudoscalar meson and a massless vector meson. The very close agreement between the present theoretically calculated value and the experimental value means that the meson exchange nucleon–nucleon interaction is one of the most realistic potentials. In these calculations, we used contributions arising from second-order and fourth-order terms and also a notice about the sixth-order terms is introduced. In the process of calculations, it appears that restricting one particle to the mass shell can be reintroduced by adding more number of terms to be included in the kernel. These new added terms to the kernel tend to cancel some of the old terms resulting in better physical agreements. In spite of some ideas before stating that the crossed-box graph cancels the uncrossed-box graph in the fourth-order terms, it appears that in other approaches the cancellation is not complete for the two fourth-order diagrams. This is very clear from the numerical agreement between the theory and the experiment in calculating the three-nucleon binding energy.

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